STATF407 Stochastic Models: second part, sheet 2

Ex 1. Consider the homogeneous Markov chain with transition matrix

$$P = \begin{bmatrix} 0 & 1/3 & 0 & 2/3 \\ 0 & 1 & 0 & 0 \\ 0 & 1/4 & 1/2 & 1/4 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

1. Draw the weighted graph associated with this Markov chain and classify the states.
2. Compute the F and R matrices (see the Lectures).

Ex 2. Consider the homogeneous Markov chain with transition matrix

$$P = \begin{bmatrix} 0.3 & 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0.4 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.8 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.4 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

1. Draw the weighted graph associated with this Markov chain and classify the states.
2. Compute the F and R matrices (see the Lectures).

Ex 2. Let (N_t) be a Poisson process with rate $\lambda > 0$. Show that if $0 < s < t$,

$$N(s)|N(t) = n$$

has a binomial distribution with parameters n and s/t.

Ex 3. Show that if N and N' are two Poisson processes with parameters λ and λ', then $N + N'$ is a Poisson process of parameter $\lambda + \lambda'$.

Ex 4. Let X_t be a standard brownian motion and fix $\theta \in$. Show that $(e^{\theta X_t - \frac{\theta^2 t}{2}})_{t \geq 0}$ is a martingale with respect to $\mathcal{A}_t := \sigma(X_u, 0 \leq u \leq t)$.